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DIOPHANTINE ANALYSIS.

133. Proposed by REV. R. D. CARMICHAEL, Hartselle, Alabama.

Find all perfect numbers of four primes and of multiplicity 4.

Solution by the PROPOSER.

The object of this note is to show that $2^5 \cdot 3^3 \cdot 5 \cdot 7 = 30240$ is the only multiply perfect number* of multiplicity 4 and having only 4 distinct primes.

Let $m = p_1^{a_1} p_2^{a_2} p_3^{a_3} p_4^{a_4}$ be the number where the primes p_1, p_2, p_3, p_4 , are in order of magnitude, beginning with the smallest. The sum of the divisors is equal to four times the number. Dividing this equation by the number, have

$$4 = \frac{p_1^{a_1+1}-1}{p_1^{a_1}(p_1-1)} \cdot \frac{p_2^{a_2+1}-1}{p_2^{a_2}(p_2-1)} \cdot \frac{p_3^{a_3+1}-1}{p_3^{a_3}(p_3-1)} \cdot \frac{p_4^{a_4+1}-1}{p_4^{a_4}(p_4-1)} \dots\dots\dots (1).$$

$$\therefore 4 < \frac{p_1}{p_1-1} \cdot \frac{p_2}{p_2-1} \cdot \frac{p_3}{p_3-1} \cdot \frac{p_4}{p_4-1} \dots\dots\dots (2), \text{ from which it may easily be shown}$$

that $p_1=2, p_2=3, p_3=5$, and $p_4=7, 11$, or 13 . From (1) we may write (for use later)

$$4 < \frac{2^{a_1+1}-1}{2^{a_1}} \cdot \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{p_4}{p_4-1} \dots\dots\dots (3).$$

Again from (1),

$$4 = \frac{2^{a_1+1}-1}{2^{a_1}} \cdot \frac{3^{a_2+1}-1}{3^{a_2} \cdot 2} \cdot \frac{5^{a_3+1}-1}{5^{a_3} \cdot 4} \cdot \frac{p_4^{a_4+1}-1}{p_4^{a_4}(p_4-1)} \dots\dots\dots (4).$$

$$\therefore 2^{a_1+5} \cdot 3^{a_2} \cdot 5^{a_3} \cdot p_4^{a_4} (p_4-1) = (2^{a_1+1}-1)(3^{a_2+1}-1)(5^{a_3+1}-1)(p_4^{a_4+1}-1) \dots\dots (5).$$

Obviously n may be so taken that

$$2^{a_1+1}-1 = (2^{2n+1}-1)(2^{2n+1}+1)(2^{2(2n+1)}+1)(2^{4(2n+1)}+1) \dots\dots\dots, \dots\dots\dots (6)$$

where limitations are to be found for the series of factors. The fourth factor introduces the prime 17, and hence not more than 3 factors may be used. We may easily show that $2^{2n+1}-1$ is not divisible by 3 or 5; and also that it is not a power of $p_4=11$ or 13 ; and that if it is a power of $p_4=7$, it is the first power. Hence, if $p_4=11$ or $13, n=0$, and we have from equation (6), $a_1=1$ or 3 . These values of p_4 and a_1 will not satisfy equation (3), however they are combined. Now, if $p_4=7, n=0$ or 1 (by the preceding). The possible values of a_1 are found from (6) to be $a_1=1, 2, 3, 5, 7$. When $a_1=7$ the prime 127 is introduced, and this value must therefore be discarded. We therefore have left to consider the cases of $p_4=7$ and $a_1=1, 2, 3$, or 5 . Substituting $p_4=7$ in equation (5) we have

*This term was introduced by D. N. Lehmer in 1901; see *Annals of Mathematics*, Ser. 2, Vol. 2, p. 103. "A multiply perfect number is one which is an exact divisor of the sum of all the divisors, the quotient being the multiplicity."

$$2^{a_1+6}.3^{a_2+1}.5^{a_3}.7^{a_4}=(2^{a_1+1}-1)(3^{a_2+1}-1)(5^{a_3+1}-1)(7^{a_4+1}-1) \dots\dots\dots(7).$$

Now n may be so taken that

$$3^{a_2+1}-1=(3^{2n+1}-1)(3^{2n+1}+1)(3^{2(2n+1)}+1)(3^{4(2n+1)}+1) \dots\dots\dots, \dots\dots\dots(8).$$

The last factor written introduces the prime 41, and therefore not more than three factors can be considered. It may easily be shown that $3^{2n+1}-1$ is not divisible by 5 or 7, and that it contains the factor 2 but once. Therefore, $3^{2n+1}-1=2$. Hence, $n=0$. Equation (8) now yields $a_2=1$ or 3.

A trial of each of the eight possible cases produced by every possible combination of the values $a_2=1$ or 3, and $a_1=1, 2, 3$, or 5, will result in finding but one number of the type here considered, namely, $2^5.3^3.5.7$.

MECHANICS.

187. Proposed by M. E. GRABER, A. M., Heidelberg University, Tiffin, Ohio.

Find the path described by a particle acted upon by a central force, the force being directly proportional to the distance of the particle.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Take coördinate axes through the center of force, and let μ^2 be the force on a particle of unit mass at the unit distance. Then the equations of motion are

$$\frac{d^2x}{dt^2}=-\mu^2r \cos \theta=-\mu^2x, \quad \frac{d^2y}{dt^2}=-\mu^2r \sin \theta=-\mu^2y.$$

Integrating the equations of motion we have

$$x=a \cos \mu t+b \sin \mu t, \quad y=c \cos \mu t+d \sin \mu t.$$

$$\therefore (cx-ay)^2+(dx-by)^2=(bc-ad)^2, \text{ which is an ellipse.}$$

Also solved by S. A. Corey, and Henry Heaton.

PROBLEMS FOR SOLUTION.

ALGEBRA.

262. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum to infinity the series $\frac{n}{(4n^2-1)^2}$, beginning with $n=1$, n being always odd.